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Optical solitons in semiconductor-doped glass fibres with saturation-type nonlinearity

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Abstract. We study soliton solutions of the nonlinear equation governing pulse propagation in semiconductor-doped glass (SDG) fibres which have saturation-type nonlinearity. The phase modulation of the pulse is nonlinear and no sech-type soliton exists in SDG fibres when high-order correction terms are taken into account. As compared with optical solitons in ordinary glass fibres, the property of the solitons discussed here is somewhat different.

There has recently been substantial interest in soliton propagation along optical fibres. This interest stems from the development of optical communication systems, optical switching, and fundamental nonlinear physics aspects. The propagation of the picosecond pulse envelope in a lossless nonlinear optical fibre is described by the nonlinear Schrödinger (NLS) equation [1–2]. The experiment was carried out on a monomode fibre of length 700 m at first [3]; for a femtosecond pulse, higher-order nonlinear phenomena are stimulated, and the NLS equation is extended to include higher-order correction terms, and replaced by the modified nonlinear Schrödinger (MNLS) equation, which supports different types of soliton and has been investigated in many papers [4–7]. In recent years, nonlinear properties of semiconductor-doped glass (SDG) fibres have been the interesting subjects in relation to switching in optical fibres and waveguides [8–10]. Because solitons show uniform phase shift over the entire waveform in the anomalous dispersion region, switching of the entire pulse can be achieved if solitons are used; otherwise, only partial switching of the pulse is possible [11]. As we know, the nonlinearity in SDG fibres is of the saturation type rather than the usual Kerr type, thus the NLS or MNLS equation is a poor approximation. Taking into account the saturation-type nonlinearity and using the slowly varying envelope approximation, Kumar [12] derived the nonlinear equation governing pulse propagation in SDG fibres. In this paper, the soliton solutions of the nonlinear equation are discussed by using the separating-variable method.

For an isotropic monomode SDG fibre with a circular cross-section, in the case of saturation-type nonlinearity, the wave equation for the fibre core can be written as

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{D}^{\mathrm{L}}}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 \boldsymbol{D}^{\mathrm{NL}}}{\partial t^2} \qquad (1)$$

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where c is the speed of light, and $\boldsymbol{D}^{\mathrm{L}}$ and $\boldsymbol{D}^{\mathrm{NL}}$ denote the linear and nonlinear parts of the electric displacement vector $\boldsymbol{D}$, with

$$\boldsymbol{D}^{\mathrm{L}}(t) = \int_0^{\infty} \epsilon(t')\boldsymbol{E}(t-t')\,\mathrm{d}t' \tag{2}$$

$$\boldsymbol{D}^{\mathrm{NL}}(t) = \epsilon_{\mathrm{s}}[\boldsymbol{E} - \boldsymbol{E}\exp(-\chi|\boldsymbol{E}|^2)]. \tag{3}$$

It should be pointed out that we use equation (3) as the mathematical model for saturation-type nonlinearity in this paper; it is due to equation (3) that the new nonlinear equation is obtained and solutions are found. In the Kerr-type medium, we have the well known relation $\boldsymbol{D}^{\mathrm{NL}} = n_2|\boldsymbol{E}|^2\boldsymbol{E}$, from which the NLS equation is derived. The above relation is an approximation of equation (3), as we can see clearly when we expand $\exp(-\chi|\boldsymbol{E}|^2)$ in a Taylor series and omit higher-order terms.

Supposing that the solitary wave is entirely supported by the HE_{11} mode of the fibre far from the cut-off, then the electric field is confined entirely in the core and its major component is the traverse one, which is linearly polarized. Hence the electric field $\boldsymbol{E}$ is assumed to be of the following form:

$$\boldsymbol{E}(x,t,\boldsymbol{r}) = \boldsymbol{e}R(\boldsymbol{r})A(t,x)\exp[-\mathrm{i}(wt-k_0x)] \tag{4}$$

where $\boldsymbol{e}$ is the unit vector in the direction of polarization, k_0 is the propagation constant, $R(\boldsymbol{r})$ is the mode function and $A(x,t)$ is the slowly varying envelope complex amplitude. Here $\boldsymbol{r}$ is a two-dimensional vector in the plane perpendicular to the x-axis.

In the slowly varying envelope approximation, one obtains the following dimensionless nonlinear equation [12]:

$$\begin{aligned}\mathrm{i}q_\xi + \frac{1}{2}q_{\tau\tau} + q\left(1 - \frac{1}{|q|^2} + \frac{\exp(-|q|^2)}{|q|^2}\right) &= \mathrm{i}\alpha q_\tau\left(1 - \frac{1}{|q|^2} + \frac{\exp(-|q|^2)}{|q|^2}\right)\\ &\quad - \mathrm{i}\alpha q(|q|^2)_\tau\left[\frac{1}{|q|^4} - \exp(-|q|^2)\left(\frac{1}{|q|^2} + \frac{1}{|q|^4}\right)\right]\end{aligned} \tag{5}$$

with

$$q = A/A_0 \qquad A_0 = 1/\sqrt{(\chi)} \tag{6}$$

$$\xi = (k_0n_2/n_0\chi)x \tag{7}$$

$$\tau = \sqrt{(k_0n_2/n_0\chi(-k_{\omega\omega}))}(t - x/v_{\mathrm{g}}) \tag{8}$$

$$\alpha = \frac{1}{v_{\mathrm{g}}}\left(\sqrt{\frac{n_2}{n_0\chi(-k_{\omega\omega})}}\right) \tag{9}$$

where $\chi = \epsilon_2/\epsilon_{\mathrm{s}}$ is the parameter representing the saturation-type nonlinearity, and the other terms have the same meanings as for ordinary glass fibres. The terms on the right-hand side of equation (5) are a higher-order correction, in the absence of which equation (5) has the following form:

$$\mathrm{i}q_\xi + \frac{1}{2}q_{\tau\tau} + q\left(1 - \frac{1}{|q|^2} + \frac{\exp(-|q|^2)}{|q|^2}\right) = 0. \tag{10}$$

In a previous letter, soliton solutions of equation (10) have been studied [13]. The main conclusions were that equation (10) contains two kinds of optical solitons: the sech type and the 'combined' type; and the phase modulation of the pulse is linear. When higher-order correction terms are taken into account, optical solitions existing in SDG fibres have different properties. It is the purpose of the present paper to look for soliton solutions of equation (5) and compare the results with those for ordinary glass fibres.

Separating $q(\xi,\tau)$ into the real amplitude $\rho(\xi,\tau)$ and phase $\phi(\xi,\tau)$, according to $A(\xi,\tau)=\rho\exp(\mathrm{i}\phi)$, and inserting it into equation (5), we obtain

$$\rho_\xi+\rho_\tau\phi_\tau+\frac{1}{2}\rho\phi_{\tau\tau}=-\alpha\rho_\tau\left(1+\frac{1}{\rho^2}-\frac{\exp(-\rho^2)}{\rho^2}-2\exp(-\rho^2)\right) \tag{11}$$

$$-\rho\phi_\xi+\frac{1}{2}\rho_{\tau\tau}-\frac{1}{2}\rho\phi_\tau^2+\rho\left(1-\frac{1}{\rho^2}+\frac{\exp(-\rho^2)}{\rho^2}\right)=\alpha\rho\phi_\tau\left(1-\frac{1}{\rho^2}+\frac{\exp(-\rho^2)}{\rho^2}\right). \tag{12}$$

In order to obtain the travelling wave soliton solution of equations (11) and (12), we look for solutions of the form

$$\rho=\rho(z) \tag{13}$$

$$\phi=\phi(z,\xi) \tag{14}$$

where $z=\tau-M\xi$ and the ξ dependence of ϕ is restricted by the conditions assumed as follows [4]:

$$\phi_\xi=\mathrm{const}=k \tag{15}$$

$$\phi_z'\text{is independent of }\xi. \tag{16}$$

The constants M and k correspond to the inverse soliton velocity shift and wavenumber shift, respectively.

Inserting equations (13) and (14) into equations (11) and (12) using equations (15) and (16), one obtains

$$-M\rho_z-\rho_z\phi_z+\frac{1}{2}\rho\phi_{zz}=-\alpha\rho_z\left(1+\frac{1}{\rho^2}-\frac{\exp(-\rho^2)}{\rho^2}-2\exp(-\rho^2)\right) \tag{17}$$

$$-\rho(k-M\phi_z)+\frac{1}{2}\rho_{zz}-\frac{1}{2}\rho\phi_z^2+\rho\left(1-\frac{1}{\rho^2}+\frac{\exp(-\rho^2)}{\rho^2}\right)=\alpha\rho\phi_z\left(1-\frac{1}{\rho^2}+\frac{\exp(-\rho^2)}{\rho^2}\right) \tag{18}$$

after multiplication with ρ, equation (17) can be integrated once to yield

$$\phi_z=M-\frac{3\alpha}{4}\rho^2+\frac{5\alpha}{18}\rho^4+\cdots+\frac{\alpha}{n+1}\left(\frac{1}{(n+1)!}-\frac{2}{n!}\right)(-1)^{n+1}\rho^{2n}+\cdots \tag{19}$$

where we consider only the case of a 'bright' soliton solution where $\rho\to0$ as $z\to\pm\infty$. Hence the integral constant in equation (19) is equal to zero.

At first, we keep terms up to ρ^2 and neglect higher-order terms in equation (19), then

$$\phi_z = M - \frac{3\alpha}{4}\rho^2. \tag{20}$$

Inserting equation (20) into equation (18) and expanding $\exp(-\rho^2)$ in a Taylor series and omitting higher-order terms in equation (18), equation (18) becomes

$$\frac{1}{2}\rho_{zz} - u^2\rho + \frac{(1-\alpha M)}{2}\rho^2 + \frac{3\alpha^2}{32}\rho^5 = 0 \tag{21}$$

where $u^2 = k - M^2/2$, equation (21) can be integrated once and transformed into a form analogous to the equation of motion of a particle in a one-dimensional potential field,

$$\tfrac{1}{2}\rho_z^2 + V(\rho) = 0 \tag{22}$$

where the potential field $V(\rho)$ is

$$V(\rho) = \frac{\alpha^2}{32}\rho^6 + \frac{(1-\alpha M)}{4}\rho^4 - u^2\rho^2. \tag{23}$$

Assuming that the peak of the pulse is located at $z = 0$, i.e. $\rho(0) = \rho_0$, and $\rho'(0) = 0$, we have $V(\rho_0) = 0$, which specifies the relation between the wavenumber shift k and the soliton peak amplitude as

$$k = \frac{M^2}{2} + \frac{(1-\alpha M)}{4}\rho_0^2 + \frac{\alpha^2}{32}\rho_0^4. \tag{24}$$

The formal solution of equation (22) is obtained according to

$$\int_{\rho_0}^{\rho} \frac{\mathrm{d}\rho}{[-2V(\rho)]^{1/2}} = \pm z \tag{25}$$

after some careful operations one can express ρ^2 as

$$\rho^2 = \frac{\rho_0^2}{2-\nu}\left(\cosh^2(\mu z) - \frac{1-\nu}{2-\nu}\right)^{-1} \tag{26}$$

with

$$\mu^2 = \frac{\alpha^2}{16}\rho_0^4 + \frac{1}{2}(1-\alpha M)\rho_0^2 \tag{27}$$

$$\nu = \frac{\rho_0^2}{2\mu^2}(1-\alpha M). \tag{28}$$

Figure 1 is a diagram of ρ versus z from equation (26); the selected parameters are $\rho_0 = 1.0$, $\mu = 0.5$ and $\nu = 0.2$. It is noted that in ordinary glass fibres, when including a nonlinear correction term involving a time derivative of the pulse envelope, the MNLS equation has a similar soliton solution to equation (26) [5], which is the basic soliton existing in SDG fibres when higher-order corrections are not neglected and for a pulse of low intensity. So, the MNLS equation is an approximate of the nonlinear equation which

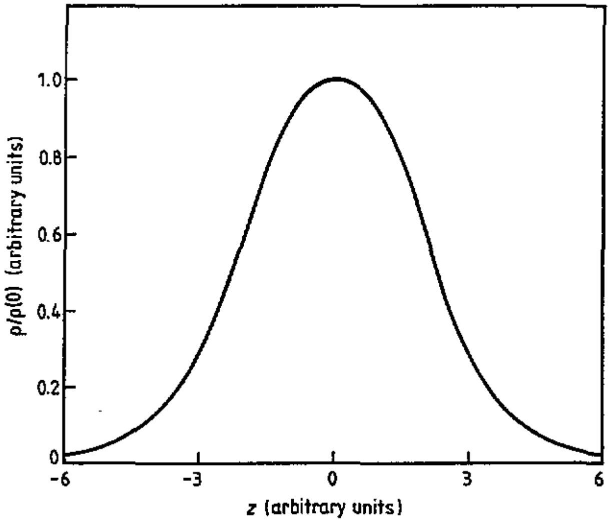

Figure 1. The curve of ρ versus z from equation (26), with the following parameters: $\rho_0 = 1.0$, $\mu = 0.5$ and $\nu = 0.2$.

describes pulse propagation in SDG fibres. In the limit when $\alpha = 0$, we have $\mu^2 = \rho_0^2/2$ and $\nu = 1$, and the soliton form of ρ is changed to a sech type. This is natural since when $\alpha = 0$, higher-order correction terms are neglected, and equation (5) identifies equation (10), which supports a sech-type soliton for a pulse of low intensity [13].

In general, we keep terms up to ρ^{2n} in equation (19) and insert it into equation (18). We then obtain an equation whose highest order is determined by $\rho\phi_z^2$; the equation can be transformed to

$$w_z^2 + V(w) = 0 \tag{29}$$

where $w = \rho^2$ and

$$V(w) = -w^2(a_0 + a_1 w + \cdots + a_{2n} w^{2n}). \tag{30}$$

Such a potential function can be written as follows:

$$V(w) = -a_{2n} w^2 (w + \lambda_1)^2 (w + \lambda_2)^2 \ldots (w + \lambda_{n-1})^2 (w^2 + \lambda_n w + \lambda_{n+1}) \tag{31}$$

where $\lambda_1, \lambda_2 \ldots \lambda_{n+1}$ are the constants determined by $a_1, a_2, \ldots a_{2n}, a_{2n}, \lambda_{n+1} > 0$, $\lambda_1, \lambda_2, \ldots \lambda_n < 0$, and $\lambda_1, < \lambda_2 < \lambda_3 < \ldots < \lambda_n$. Hence, equations (29) and (31) are integrable and support a 'combined'-type soliton solution, and their general form can be found as [14]

$$\rho^2 = \frac{2\lambda_{n+1}}{\sqrt{\lambda_n^2 - 4\lambda}} \frac{2}{\exp(X) - 2\lambda_n/\sqrt{\lambda_n^2 - 4\lambda_{n+1}} + \exp(-X)} \tag{32}$$

where

$$X = \sqrt{\lambda_{n+1}} \left(\frac{\sqrt{a_{2n}}}{b_0} z + G(F_1(z)F_2(z), \ldots, F_{n-1}(z)) \right) \tag{33}$$

$$G(F_1(z), F_2(z), \ldots, F_{n-1}(z)) = \frac{1}{b_0}\sum_{m=1}^{n-1}\frac{b_m}{\sqrt{Y}} \times \ln\left|\frac{\sqrt{F_m^2(z) + (-2\lambda_m + \lambda_n)F_m(z) + Y} + \sqrt{Y}}{F_m(z)} + \frac{(-2\lambda_m + \lambda_n)}{\sqrt{Y}}\right| \tag{34}$$

with

$$Y = \lambda_m^2 - \lambda_m\lambda_n + \lambda_{n+1} > 0$$

$$F_m(z) = \rho^2(z) + \lambda_m$$

$$b_0 = (\lambda_1\lambda_2 \ldots \lambda_{n-1})^{-1}$$

$$b_m = \left[\frac{\rho^2 + \lambda_m}{\rho^2(\rho^2 + \lambda_1)\ldots(\rho^2 + \lambda_{n-1})}\right]_{\rho^2 = -\lambda_m} \qquad m = 1, 2, \ldots n - 1. \tag{35}$$

From the above discussion, we notice that when the higher-order correction terms are included in the nonlinear equation, i.e. for a femtosecond pulse, there are two kinds of solitons existing in SDG fibres. One is the cosh type, and the other the 'combined' type. The first type corresponds to a low intensity of the pulse envelope, where terms higher than ρ^6 can be neglected in the potential function, and it is the lowest approximate soliton solution of equation (5) and the basic soliton existing in SDG fibres when α cannot be neglected. As for the second type of soliton, it should be stressed that for each n there is a 'combined' shape soliton and each of these solitons is the nth-order approximate solution of equations (11) and (12). Theoretically, the exact soliton solution of equations (11) and (12) is the 'combined' form when $n \to \infty$. However, the coefficients in the expansion of equation (19) become less and less when the value of n increases. Then for the finite intensity of the optical pulse envelope, a limited nth-order soliton is a good approximate for the exact solution. To make the description clearer, we show equation (32) diagrammatically in figure 2 for $n = 2$, $\lambda_1 = -4$, $\lambda_2 = -0.9$, $\lambda_3 = 0.2$ and $a_4 = 1$, and plot curves of equation (32) for $n = 3, 4$, with other parameters as shown in the figure caption. From figure 3, we can see that the two curves almost overlap, in other words they tend to have the same form, which may be regarded as the soliton propagating in SDG optical fibres. In our numerical simulations, we find that the parameters may be different from each other; however, the soliton existing in SDG fibres can be represented by a limited nth-order approximate solution which has the 'combined' form as expressed in equation (32).

Kothari [15] investigated the nonlinear composite medium. He concluded that effective nonlinearity in a composite material must have saturation, and for a low-intensity field the effective-medium dielectric constant ϵ_{eff} can be expanded into a power series of $|E|^2$,

$$\epsilon_{\text{eff}} = \epsilon + 4\pi\chi^{(3)}|E|^2 + 4\pi\chi^{(5)}|E|^4 + \cdots \tag{36}$$

so, for a low-intensity field in saturation-type optical fibres, the pulse propagation is similar to that for the high-intensity field in ordinary optical fibres where the higher-order nonlinear refraction indices should be included. But the physical mechanism is somewhat different in two cases.

In conclusion, for weak saturation-type nonlinearity SDG fibres, the NLS (or MNLS) equation is a good approximate nonlinear equation which governs picosecond (or femtosecond) pulse propagation as in the case for ordinary glass fibres. In general, the optical soliton propagating in SDG fibres is the 'combined' type, which is not supported in ordinary glass fibres.

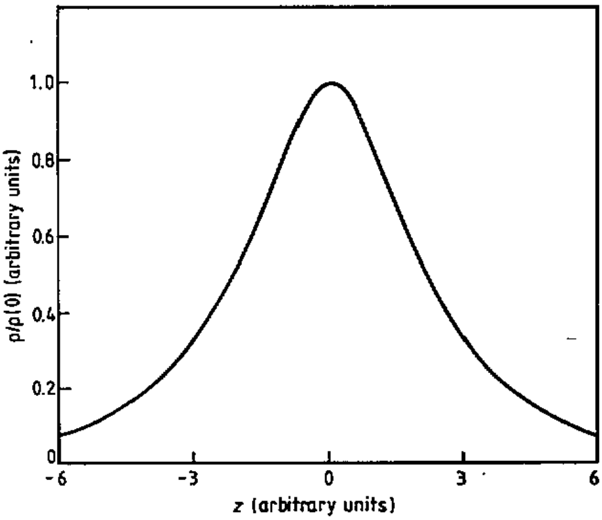

Figure 2. The soliton shape of the 'combined'-type solution for $n = 2$. The other parameters are as follows: $\lambda_1 = -4.0$, $\lambda_2 = -0.9$, $\lambda_3 = 0.2$ and $a_4 = 1.0$.

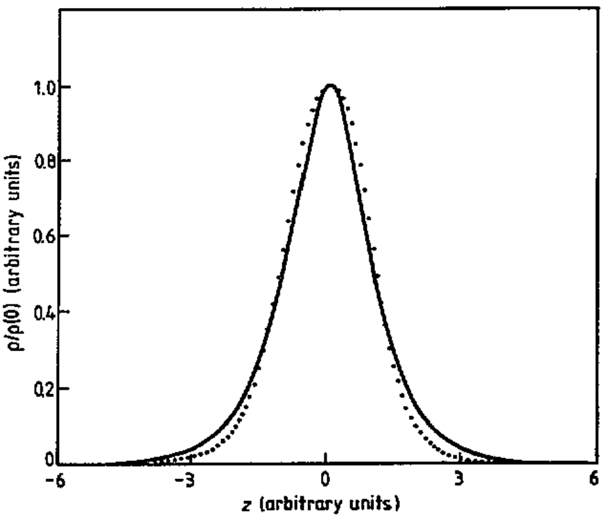

Figure 3. The soliton curves for $n = 3$ and $n = 4$, where the full (broken) line corresponds to $n = 3$ ($n = 4$), respectively. The parameters are as follows: $n = 3, \lambda_1 = -4.0, \lambda_2 = -2.0, \lambda_3 = -0.9, \lambda_4 = 0.2, a_6 = 0.5$ and $n = 4, \lambda_1 = -4.0, \lambda_2 = -2.0, \lambda_3 = -1.5, \lambda_4 = -0.9, \lambda_5 = 0.2, a_8 = 0.4$.

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